KNT/KW/16/5154

Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

STATISTICS (Statistical Inference)

Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :— All the **FIVE** questions are compulsory and carry equal marks.

1. (A) When is the estimator said to be unbiased for the parameter ? If a random sample of size n is taken from an infinite population and if $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$, then show that S^2 is an unbiased estimator for the population variance σ^2.

(B) Define :

(i) Critical region

(ii) Type I error and type II error.

If X has a p.d.f.

$$f(x) = \begin{cases} (1+\theta)^{x^{\theta}}, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$ the critical region is $\{X \ge 0.8\}$. Calculate sizes of type I error, type II error and power of the test. 5+5

OR

(E) Define :

(i) Null and alternative hypothesis

(ii) Two-tailed and one-tailed test

(iii) Level of significance and p-value.

(F) A random sample of size n is drawn from a Bernoulli population with parameter p. Show that sample mean is unbiased estimator of p. Also, find its standard error.

(G) Define :

(i) UMVUE

(ii) Efficiency of an estimator

(iii) Confidence limits and degree of confidence.

(H) If T is an unbiased estimator of θ, then show that :

(i) aT + b is an unbiased estimator of aθ + b where a and b are constants.

(ii) T^2 is biased estimator of θ^2. 2½+2½+2½+2½

2. (A) Explain t-test for testing :

(i) the significance of a sample mean

(ii) the significance of the difference between two sample means based on two independent samples.

State the assumptions clearly. Also, construct 100 (1 – α)% confidence interval for

(i) population mean μ

(ii) difference of two population means for the above samples. 10

OR

(E) Explain F-test for equality of population variances when the population means are unknown stating the assumptions. Also estimate 100 (1 – α)% confidence interval for the ratio of two population variances.

(F) Describe paired t-test. Also estimate 100 (1 – α)% confidence interval for the difference of means. 5+5

3. (A) Describe the test for testing specified value of variance of a normal population. Also construct 100 (1 – α)% confidence interval for population variance. [Assume that population mean is unknown].

(B) Describe chi-square test for testing goodness of Fit. 5+5

OR

(E) Explain chi-square test for testing the independence of attributes in 2 × k contingency table. Also derive Brandt-Snedecer formula for chi-square in this case.

(F) Describe the chi-square test for homogeneity of populations. 5+5

4. (A) Explain the use of central limit theorem in testing of hypothesis. Describe the test for testing the specified value of single population proportion when sample size is large. Also, state 100 (1 – α)% confidence interval for population proportion.

(B) Explain large sample test for testing the significance of the difference of two population means. Also discuss the cases when

(i) $H_1 : \mu_1 > \mu_2$

(ii) $H_1 : \mu_1 < \mu_2$ 5+5

OR

(E) Explain large sample test for testing the specified value of a single population mean. Construct 100 (1 –α)% confidence interval for it. Assume that population variance is unknown.

(F) To test $H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2$ describe a test based on large samples. Also construct 100 $(1 - \alpha)$% confidence interval for the difference of two population proportions using the above samples. 5+5

5. Solve any **TEN** questions from the following :—

(A) State Cramer-Rao inequality.

(B) Prove or disprove an unbiased estimator need not be unique.

(C) Who developed the chi-square test for goodness of fit ?

(D) State R-command for goodness of fit test.

(E) When is Yates correction for continuity applied ?

(F) State R-command for paired t-test.

(G) State the test statistic for testing the significance of sample correlation coefficient in sampling from bivariate normal population.

(H) Using a graph, show that, $t_{n,(1-\alpha)} = -t_{n,\alpha}$.

(I) State true or false :

''If the null hypothesis is accepted at 5% level of significance, it will be accepted at 1% level of significance.'' Justify your answer.

(J) State the 100 $(1 - \alpha)$% confidence interval for the difference of two population means on the basis of two large samples, assuming unknown variances.

(K) To test $H_0 : \mu = 2200$ against $H_1 : \mu \neq 2200$, given a sample of size 400 has mean 2250 and s.d. 12, calculate the value of test statistic to be used.

(L) A sample of 3000 persons was selected from a city. In the sample there were 1632 males. Which test is used to check whether this information supports the view that the number of males is equal to the number of females in the city ? 1×10=10